Name _____ Test 1, Fall 2020

1) Multiply the two matrices below or state why they cannot be multiplied. (15 points)

$$\begin{bmatrix} 1 & 4 \\ 2 & 3 \\ 5 & -1 \end{bmatrix} \begin{bmatrix} 0 & -3 & 1 \\ 2 & 2 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 0+8 & -3+8 & 1+16 \\ 0+6 & -6+6 & 2+12 \\ 0-2 & -15-2 & 5-4 \end{bmatrix} = \begin{bmatrix} 8 & 5 & 17 \\ 6 & 0 & 14 \\ -2 & -17 & 1 \end{bmatrix}$$

2) Find the null space of the matrix below. (16 points)

$$\begin{bmatrix}1 & 1 & 3\\ 2 & 2 & 6\\ 0 & 0 & 4\\ 0 & 0 & 7\\ 0 & 0 & 0\end{bmatrix} \sim_R \begin{bmatrix}1 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$x + y = 0$
$x = -y$

$z = 0$

$$\left\{\begin{bmatrix}-y\\ y\\ 0\end{bmatrix} : y \in \mathbb{R}\right\} = \left\{\begin{bmatrix}-1\\ 1\\ 0\end{bmatrix} y : y \in \mathbb{R}\right\} = span\left(\left\{\begin{bmatrix}-1\\ 1\\ 0\end{bmatrix}\right\}\right)$$

3) Reduce the matrix below to reduced row echelon form. (16 points)

$$\begin{bmatrix} 1 & 2 & 5 & 6 \\ 0 & 2 & 4 & 6 \\ 2 & 4 & 10 & 15 \\ 1 & 4 & 9 & 12 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 5 & 6 \\ 0 & 2 & 4 & 6 \\ 2 & 4 & 10 & 15 \\ 1 & 4 & 9 & 12 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 2 & 5 & 6 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & 0 & 3 \\ 1 & 4 & 9 & 12 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 2 & 5 & 6 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & 0 & 3 \\ 0 & 2 & 4 & 6 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 2 & 5 & 6 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 2 & 5 & 6 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 \to R_3 - 2R_1$ $R_4 \to R_4 - R_1$ $R_4 \to R_4 - 2R_2$ $R_2 \to \frac{1}{2}R_2$

$$\sim_R \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 \to R_1 - 2R_2$ $R_3 \to \frac{1}{3}R_3$ $R_2 \to R_2 - 3R_3$

4) Answer the questions below (3 points each)

(A) Let A be a 2×4 matrix. How many solutions does $A\vec{x} = \vec{0}$ have?

∞

(B) Let A be a 4×2 matrix. In row reduced echelon form, it has 3 rows of zeroes. How many solutions does $A\vec{x} = \vec{0}$ have?

∞

(C) Let A be a 3×3 matrix such that $A\vec{x} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ has one solution. How many solutions does $A\vec{x} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$ have?

1

(D) If A is a 7×7 matrix and the dimension of the row space is 5, what is the dimension of the column space?

5

(E) If A is a 7×5 matrix such that $A\vec{x} = [1 \quad 1 \quad 1 \quad 1 \quad 1]^T$ has infinitely many solutions, what is the maximum dimension of the row space of A?

4

5) Find the length of the vector below. (8 points)

$$\begin{bmatrix} 1 \\ 2 \\ 0 \\ 2 \end{bmatrix}$$

$\sqrt{1+4+4} = \sqrt{9} = 3$

6) Are the vectors below orthogonal to each other? Justify your answer. (8 points)

$$\begin{bmatrix}1\\2\\0\\4\end{bmatrix} \text{ and } \begin{bmatrix}-2\\3\\5\\0\end{bmatrix}$$

$$\begin{bmatrix}1\\2\\0\\4\end{bmatrix} \bullet \begin{bmatrix}-2\\3\\5\\0\end{bmatrix} = -2 + 6 + 0 + 0 = 4 \neq 0$$

They are not orthogonal, as the above dot product is nonzero.

7) Identify a good partition to use to multiply the matrices below, then multiply them. (7 points)

$$\left[\begin{array}{cc|ccccc} 1 & 2 & 0 & 0 & 0 & 0 & 0 \\ 3 & 4 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]\left[\begin{array}{cc|cc} 2 & -1 & 0 & 0 \\ 5 & 3 & 0 & 0 \\ \hline 0 & 0 & 4 & -5 \\ 0 & 0 & 7 & 7 \\ 0 & 0 & 9 & 11 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 4 & 6 \end{array}\right]$$

$$\begin{bmatrix} A & \vec{0} \\ \vec{0} & I \end{bmatrix}\begin{bmatrix} B & \vec{0} \\ \vec{0} & C \end{bmatrix} = \begin{bmatrix} AB & \vec{0} \\ \vec{0} & C \end{bmatrix}$$

$$\begin{bmatrix} 12 & 5 & 0 & 0 \\ 26 & 9 & 0 & 0 \\ 0 & 0 & 4 & -5 \\ 0 & 0 & 7 & 7 \\ 0 & 0 & 9 & 11 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 4 & 6 \end{bmatrix}$$

8) Multiply the vector $\vec{v} = \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$ by the scalar 5. (8 points)

$$\begin{bmatrix} 5 \\ 20 \\ 15 \end{bmatrix}$$

9) Add the matrices below. (7 points)

$$\begin{bmatrix} 2 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 3 & -3 \\ 0 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 5 & -1 \\ 3 & 8 \end{bmatrix}$$